| | | | |
|---|---|---|---|
| 1) | | | |
| a) | $\mathrm{P}(A > 2B) = \mathrm{P}(A - 2B > 0)$<br>Let $X = A - 2B$ $\quad X \sim \mathrm{N}(-2,\ 49)$ | M1<br>M1<br>A1 | 2.1<br>3.3<br>1.1b |
| | $\mathrm{P}(X > 0) = 0.387548 \ldots$<br>awrt 0.388 | A1 | 1.1b |
| | | | **(4)** |
| b) | $\mathrm{Var}(T) = n\mathrm{Var}(A) + n^2\mathrm{Var}(B)$<br>$\mathrm{Var}(T) = 9n + 10n^2$ | M1<br>A1 | 3.4<br>1.1b |
| | $10n^2 + 9n = 196$ or $10n^2 + 9n - 196 = 0$<br>From calculator $n = 4$ or $n = -4.9$<br>Positive integer so $n = 4$ | M1<br><br>A1 | 1.1b<br><br>2.2a |
| | | | **(4)** |
| | | | **(8)** |

Notes

a)

1st M1 Use of $A - 2B > 0$ or $B - 2A < 0$

2nd M1 normal distribution with mean = -2 or 2

A1 distribution of *X* all correct (may be implied by later working)

2nd A1 (probably from calculator) awrt 0.388

b)

1st M1 finding expression for Var(*T*) at least one term correct

1st A1 Var(*T*) all correct

2nd M1 quadratic equation formed using their Var(*T*)

2nd A1 correct answer, giving $n = 4$ only.

| | | | |
|---|---|---|---|
| 2) | | | |
| a) | $s_A{}^2 = 3.8027777\ldots$ awrt $s_A{}^2 = 3.80$<br>$s_B{}^2 = \frac{13}{12}\left(\frac{106800}{13} - \left(\frac{1175}{13}\right)^2\right) = 49.83974359$<br>awrt $s_B{}^2 = 49.8$ | B1<br>M1<br>A1 | 1.1b<br>1.1b<br>1.1b |
| | Testing $\mathrm{H}_0: \sigma_A{}^2 = \sigma_B{}^2$ against $\mathrm{H}_1: \sigma_A{}^2 \neq \sigma_B{}^2$ | B1 | 2.5 |
| | Critical value $F_{12,8} = 5.67$<br>Test statistic $F = \frac{'49.8397\ldots'}{'3.8027\ldots'} = 13.106\ldots$ | B1<br>M1 | 1.2<br>3.4 |
| | Test statistic is in critical region, sufficient evidence to reject $\mathrm{H}_0$, evidence that the **variances** of the maximum power outputs of the cars from the **two production lines** may be **different**. oe | A1cso | 2.2b |
| | | | **(7)** |
| b) | Testing $\mathrm{H}_0: \sigma_B = 3.5$ against $\mathrm{H}_1: \sigma_B > 3.5$<br>Critical value $\chi^2_{v=12} = 21.026$<br>Test statistic $\chi^2 = \frac{12 \times 49.839\ldots}{3.5^2} = 48.822\ldots$ | B1<br>B1<br>M1 | 2.5<br>1.2<br>3.4 |
| | Test statistic is in critical region, sufficient evidence to reject $\mathrm{H}_0$, evidence that the **standard deviation** of the maximum **power output** of cars from **production line *B*** is **too high**. | A1cso | 2.2b |
| | | | **(4)** |
| | | | **(11)** |

Notes

a)

1st B1 from calculator or otherwise awrt 3.80 (accept *s* = awrt 1.95)

1st M1 correct use of formula with given statistics

1st A1 awrt 49.8 (accept *s* = 7.06)

2nd B1 both hypotheses correct in terms of $\sigma$ (not *s*), accept $\mathrm{H}_0: \sigma_A = \sigma_B$ and $\mathrm{H}_1: \sigma_A \neq \sigma_B$

3rd B1 awrt 5.67 (or 0.222)

2nd M1 test statistic correct using their variances (but not standard deviations)

(may use $F = \frac{'3.8027\ldots'}{'49.8397\ldots'} = 0.07630\ldots$ but only if consistent with their critical value.)

2nd A1cso correct conclusion given <u>in context.</u>

b)

1st B1 both hypotheses correct in terms of $\sigma$ (not *s*), accept $\mathrm{H}_0: \sigma_B{}^2 = 12.25$, $\mathrm{H}_1: \sigma_B{}^2 > 12.25$ oe

2nd B1 awrt 21.0

1st M1 ft their $s_B$

1st A1cso correct conclusion in context, must include **standard deviation** or **variance** and **power output** or **production line *B*** and **too high** (oe)

<table>
<tr><td>3)</td><td></td><td></td><td></td></tr>
<tr><td>a)</td><td><table>
<tr><td>Jam</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td></tr>
<tr><td>Ranks, Judge 1</td><td>4.5</td><td>4.5</td><td>1</td><td>6</td><td>7</td><td>2</td><td>8</td><td>3</td></tr>
<tr><td>Ranks, Judge 2</td><td>7</td><td>3</td><td>2</td><td>6</td><td>5</td><td>4</td><td>8</td><td>1</td></tr>
</table></td><td>M1<br>A1<br>A1</td><td>2.1<br>1.2<br>1.1b</td></tr>
<tr><td></td><td>$r_S$ = pmcc of ranks = 0.742528…<br>awrt 0.743</td><td>M1<br>A1</td><td>3.1b<br>1.1b</td></tr>
<tr><td></td><td></td><td></td><td>(5)</td></tr>
<tr><td>b)</td><td>Because only the order is meaningful not the points</td><td>B1</td><td>2.4</td></tr>
<tr><td></td><td></td><td></td><td>(1)</td></tr>
<tr><td>c)</td><td><table>
<tr><td>Jam</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td></tr>
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<tr><td>Ranks, Judge 3</td><td></td><td>3</td><td>2</td><td>6</td><td></td><td>1</td><td>5</td><td>4</td></tr>
<tr><td>$\boldsymbol{d^2}$</td><td></td><td>0</td><td>0</td><td>0</td><td></td><td>9</td><td>9</td><td>9</td></tr>
</table></td><td>M1</td><td>1.1b</td></tr>
<tr><td></td><td>$$1-\frac{6\times\sum d^2}{8\times 63}=\frac{4}{7}$$<br>$$\sum d^2=36$$<br>$$\therefore {d_A}^2+{d_E}^2=9$$<br>(If A is 8th and E is 7th then ${d_A}^2+{d_E}^2=1+4=5$)<br>If A is 7th and E is 8th then ${d_A}^2+{d_E}^2=0+9=9$<br>So E is awarded lowest points by judge 3</td><td>M1<br>A1<br>M1<br>A1cso</td><td>2.1<br>1.1b<br>1.1b<br>2.2a</td></tr>
<tr><td></td><td></td><td></td><td>(5)</td></tr>
<tr><td></td><td></td><td></td><td>(11)</td></tr>
</table>

Notes

a) 1st M1 attempt to rank both sets of data (**both** high to low or low to high)

1st A1 ranks of judge 1 all correct (4.5, 4.5, 8, 3, 2, 7, 1, 6 if low to high)

2nd A1 ranks of judge 2 all correct (2, 6, 7, 3, 4, 5, 1, 8 if low to high)

2nd M1 use of pmcc of ranks (calc or pmcc formula), may be implied by answer or statement or working

Note use of Spearman's formula with tied ranks is M0

3rd A1 awrt 0.743

b) B1 need reference to the ordinal nature of the data

c) 1st M1 attempt to rank available points for judge 3 (must be consistent order with their judge 2 ranks)

2nd M1 use of Spearman's rank formula

1st A1 $\sum d^2 = 36$, may be implied by later working

3rd M1 complete method to identify ranks of A and E

2nd A1cso correct answer with justification.

| | | | |
|---|---|---|---|
| 4) | | | |
| a) | $\int_0^1 kx(1-x^2)\mathrm{d}x = 1$ | M1 | 2.1 |
| | $k\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_0^1 = 1$ | A1 | 1.1b |

| | | | |
|---|---|---|---|
| | $0.25k = 1$<br>$k = 4$ | M1<br>A1 | 1.1b<br>1.1b |
| | | | **(4)** |
| b) | $\frac{\mathrm{df}}{\mathrm{d}x} = k(1 - 3x^2) = 0$<br>$x^2 = \frac{1}{3}$ $x = \frac{\sqrt{3}}{3} = 0.5773 \ldots ..$<br>awrt $x = 0.577$ | M1<br>A1 | 3.1a<br>1.1b |
| | | | **(2)** |
| c) | H(1) = 1 $\frac{\ln 2}{\alpha} = 1$<br>$\ln 2 = \alpha$ * | M1<br>A1cso* | 2.1<br>1.1b |
| | | | **(2)** |
| d) | Pdf, $\mathrm{h}(x) = \frac{\mathrm{dH}(x)}{\mathrm{d}x} = \frac{1}{\ln 2} \times \frac{1}{(1+x)}$ | M1<br>A1 | 2.1<br>1.1b |
| | $\mathrm{E}(X) = \frac{1}{\ln 2}\int_0^1 \frac{x}{1+x}\mathrm{d}x$ | M1 | 3.1a |
| | Using substitution $u = 1 + x$<br>$\therefore x = u - 1$ and $\frac{\mathrm{d}u}{\mathrm{d}x} = 1$ | M1 | 2.1 |
| | $\mathrm{E}(X) = \frac{1}{\ln 2}\int_1^2 \frac{u-1}{u}\mathrm{d}u = \frac{1}{\ln 2}\int_1^2 1 - \frac{1}{u}\mathrm{d}u$ | A1 | 1.1b |
| | $= \frac{1}{\ln 2}[u - \ln u]_{u=1}^{u=2}$ | A1 | 1.1b |
| | $= \frac{1 - \ln 2}{\ln 2}$ oe | A1 | 1.1b |
| | | | **(7)** |
| e) | $y = \mathrm{f}(x)$ has (slight) negative skew implied by mode > 0.5 or graph sketch | B1 | 2.4 |
| | $y = \mathrm{h}(x)$ has positive skew implied by mean < 0.5 or graph sketch (decreasing function) | B1 | 2.4 |
| | | | **(2)** |
| | | | **(17)** |

Notes

a) 1st M1 use of $\int \mathrm{f}(x)\mathrm{d}x = 1$

1st A1 correct integration (condone missing limits but not incorrect limits)

2nd M1 setting up and solving an equation for *k*

2nd A1 cao

b) M1 use of $\frac{\mathrm{df}}{\mathrm{d}x} = 0$

A1 $x = \frac{\sqrt{3}}{3}$ oe

c) M1 Use of H(1) = 1

A1 cso *

d) 1st M1 use of differentiation to find h(*x*)

1st A1 h(*x*) all correct

2nd M1 $\mathrm{E}(X) = \int x \times \textit{their}\ \mathrm{h}(x)\mathrm{d}x$

3rd M1 use of substitution or equivalent e.g. alt method, splitting fraction

2nd A1 correct use of subs or alt e.g.

$$\mathrm{E}(X) = \frac{1}{\ln 2}\int_0^1 \frac{1+x-1}{1+x}\mathrm{d}x = \frac{1}{\ln 2}\int_0^1 1 - \frac{1}{1+x}\mathrm{d}x$$

3rd A1 integration correct

4th A1 cao oe must be exact (eg $\frac{1}{\ln 2} - 1$)

e) 1st B1 correct answer and supporting reason for skew of $y = \mathrm{f}(x)$, accept approximately symmetrical if supported by e.g. appropriate sketch.

2nd B1 correct answer and supporting reason for skew of $y = \mathrm{h}(x)$

| 5) | | | |
|---|---|---|---|
| | Test statistic, $z = \frac{6-4}{\sqrt{\frac{16}{2n}+\frac{9}{n}}} = \frac{2}{\sqrt{\frac{17}{n}}}$ | M1<br>A1 | 3.4<br>1.1b |
| | Critical value = 1.6449 | B1 | 1.1b |
| | $\frac{2}{\sqrt{\frac{17}{n}}} > 1.6449$<br>$n > \frac{17 \times 1.6449^2}{2^2} = 11.499\ldots$<br>*n is integer*, $\therefore n \geq 12$ | M1<br>A1<br>A1cso | 2.1<br>1.1b<br>2.2a |
| | | | **(6)** |

Notes

1st M1 use of correct formula for test statistic, condone use of 4 rather than 16 and 3 rather than 9, accept + or –

1st A1 all correct, including sign

B1 1.64 or better

2nd M1 complete method (equation or inequality, either way round) solving for *n*

2nd A1 11.4 or 11.5 or better

3rd A1 cso

| | | | |
|---|---|---|---|
| 6) | | | |
| a) | $25.8 = \frac{(n_x - 1) \times 24 + (n_y - 1) \times 27}{n_x + n_y - 2}$<br><br>$25.8 = \frac{(n_x - 1) \times 24 + (n_x + 3) \times 27}{2n_x + 2}$ | M1<br><br>A1 | 2.1<br><br>1.1b |
| | $51.6n_x + 51.6 = 51n_x + 57$<br>$0.6n_x = 5.4$<br>$n_x = 9$* | M1<br><br>A1cso* | 1.1b<br><br>2.2a |
| | | | **(4)** |
| b) | Testing $\mathrm{H_0}: \mu_y = \mu_x + 2$ against $\mathrm{H_1}: \mu_y > \mu_x + 2$ (oe) | B1 | 2.5 |
| | Test statistic, $t = \frac{81.2 - 76.5 - 2}{\sqrt{25.8\left(\frac{1}{9} + \frac{1}{13}\right)}}$<br>$= 1.2258448 \ldots$ awrt 1.23 | M1<br><br>A1 | 3.1b<br><br>1.1b |
| | Critical value, $t_{20} = 1.32534 \ldots$ | B1 | 1.1b |
| | Test statistic is not in the critical region, so insufficient evidence to reject $\mathrm{H_0}$, no significant evidence to support the researcher's belief. oe | A1 | 2.2b |
| | | | **(5)** |
| | | | **(9)** |

Notes

a) M1 use of formula for pooled estimate of variance

A1 all correct including substitution for $n_x$ or $n_y$

M1 Solving a linear equation in $n_x$

A1cso* complete solution with working shown

b) 1st B1 both correct and in terms of $\mu$ not $\bar{x}$

M1 use of correct formula, condone missing 2

1st A1 awrt (-)1.23

2nd B1 awrt (-)1.33, sign must match test statistic.

A1 correct conclusion in context

| | | | |
|---|---|---|---|
| 7) | | | |
| a) | Underlying distribution of pressures at which balloons burst is **Normal** | B1 | 1.2 |
| | | | **(1)** |
| b) | (From calculator)<br>$\bar{x} = 1739.857143$<br>$s^2 = 630.8095238$ | <br>B1<br>B1 | <br>1.1b<br>1.1b |
| | $-1.943 < \dfrac{'1739.857..' - \mu}{\sqrt{\dfrac{'630.8095 \ldots'}{7}}} < 1.943$ | M1<br>A1 | 3.4<br>1.1b |
| | $1721.412 \ldots < \mu < 1758.3018 \ldots$<br>awrt (1721, 1758) | A1 | 1.1b |
| | | | **(5)** |
| c) | $1741.09 - 1728.91 = 2 \times z \times \dfrac{22}{\sqrt{40}}$ | M1<br>A1 | 2.1<br>3.4 |
| | $z = 1.75075 \ldots$ | A1 | 1.1b |
| | $\mathrm{P}(Z < 1.75075 \ldots) = 0.9600 \ldots$<br>So 4% in each tail, so 92% CI awrt $k = 92$ | M1<br>A1 | 1.1b<br>2.2a |
| | | | **(5)** |
| d) | P(all 5 C.I.s contain $\mu$) = '0.92'$^5$<br>=0.65908…<br>awrt 0.659 | M1<br>A1 | 3.1b<br>1.1b |
| | | | **(2)** |
| | | | **(13)** |

Notes

a) B1 comment in context, must mention Normal distribution

b) 1st B1 awrt 1740

2nd B1 awrt 631 (or $s$ = awrt 25.1)

M1 use of correct formula

1st A1 all correct with $t$ values of 1.94 or better

2nd A1 awrt 1721 and 1758 (both correct)

c) 1st M1 use of $(2) \times z$ or $t \times \frac{22}{\sqrt{40}}$

1st A1 fully correct equation with use of $z$ not $t$

2nd A1 $z$ = awrt 1.75

2nd M1 use of inverse normal (may be implied by answer)

3rd A1 depend on M2

d) M1 $p^5$

A1 awrt 0.659